Numeral *any*: in favor of viability*

Luis Alonso-Ovalle & Jonathan Palucci

McGill University

1. Introduction

The distribution of DPs headed by *any* is sensitive to whether the NP contains a numeral. Outside downward entailing environments, *any* DPs are licensed in sentences containing a possibility modal, (1b), but deviant in positive episodic sentences, as seen in (1a). In sentences containing a necessity modal, as in (1c), only *any* DPs containing NPs modified by numerals ('numeral *any*') are licensed (Dayal 2005, 2013, Chierchia 2013).

(1) a. * Bill read any book. / * Bill read any two books.
b. Bill can read any book. / Bill can read any two books.
c. * Bill must read any book / Bill must read any two books.

The paper reviews in Section 2 two competing analyses of the contrast between *any* and numeral *any*—the Wide Scope Constraint Analysis (WSA) (Chierchia 2013) and the Viability Constraint Analysis (VCA) (Dayal 2013)—and shows in Section 3 that the WSA overgenerates interpretations for sentences containing collective predicates.

2. Two analyses of the contrast between *any* and numeral *any*

2.1 The Wide Scope Constraint Analysis (Chierchia 2013)

Chierchia (2013) analyzes free choice items as existential quantifiers. The LF for the first sentence in (1a), for instance, contains (2), which conveys that Bill read at least one book.[1]

*Thanks to the reviewers and conference participants, and to Esmail Moghiseh for insightful discussion. This project was funded by the Social Sciences and Humanities Research Council of Canada (*Modality across Categories*, 435-2018-0524, PI: Alonso-Ovalle). Our names are listed in alphabetical order.

[1]We assume an interpretation function relativized to a world (and a variable assignment), mapping IPs to truth values, and use $[\![\alpha]\!]$ (for α of type t) to refer to $\lambda w.[\![\alpha]\!]^w$). 'R(a)' stands for the function named by '$\lambda w.\textsc{read}_w(\text{Bill})(a)$', and '$R(a) \vee R(b)$' for '$\lambda w.\textsc{read}_w(\text{Bill})(a) \vee \textsc{read}_w(\text{Bill})(b)$'—likewise for conjunction. '$[\![\alpha]\!]^{\text{ALT}-x}$' refers to the set of alternatives to $[\![\alpha]\!]$ of type x.

(2) $[\![\text{any book}_D\ \lambda_1\ \text{Bill read } t_1]\!]^g = R(a) \vee R(b)$ $\qquad$ $(g(D)= [\![\text{book}]\!]^w = \{a, b\})$

On top of this, *any* DPs trigger two types of alternatives that end up being propositional ('pre-exhaustified' domain alternatives — PDAs — and scalar alternatives). Two types of exhaustification operators ($O_{\text{EXH-D}}$ and O_σ) range over these alternatives, as in (3).

(3) $[\![O_x\ \phi]\!] = \lambda w.[\![\phi]\!](w) = 1 \wedge \forall p \in [\![\phi]\!]^{\text{ALT}-x}[p(w) = 1 \rightarrow [\![\phi]\!] \subseteq p]$

In positive episodic sentences, exhaustification yields a contradiction. Consider (4). The set of scalar alternatives to the argument of O_σ is (5a). O_σ excludes the proposition in (5a), deriving (6a). The domain alternatives to the argument of $O_{\text{EXH-D}}$, in (5b), correspond to the proposition that this argument expresses when the domain of quantification of *any* is restricted to any subset of its original domain. The set of pre-exhaustified domain alternatives, in (5c), is the set containing for any domain alternative p, the result of strengthening p with the exclusion of any other proposition in the set of domain alternatives that is 'innocently excludable.'[2] Excluding both alternatives in (5c) derives for (4) a contradiction, as in (6b).

(4) $O_{\text{EXH-D}}\ O_\sigma$ any book$_D$ λ_1 Bill read t_1

(5) a. $\{R(a) \wedge R(b)\}$ (scalar alternative)
b. $\{R(a), R(b)\}$ (domain alternatives)
c. $\{R(a) \wedge \neg R(b), R(b) \wedge \neg R(a)\}$ (pre-exhaustified domain alternatives)
d. $R(a) \leftrightarrow R(b)$ (domain implicature)

(6) a. $[\![O_\sigma\ \text{any book}_D\ \lambda_1\ \text{Bill read } t_1]\!] = [R(a) \vee R(b)] \wedge \neg[R(a) \wedge R(b)]$
b. $[\![O_{\text{EXH-D}}\ O_\sigma\ \text{any book}_D\ \lambda_1\ \text{Bill read } t_1]\!] = (6a) \wedge [R(a) \leftrightarrow R(b)](\Leftrightarrow \bot)$

The WSA assumes that *any* must scope over modals, and, so, exhaustification also yields a contradiction in modal sentences containing *any*, as seen in (7a) and (7b).

(7) a. $[\![O_{\text{EXH-D}}\ O_\sigma\ \text{any book}_D\ \lambda_1\ \text{must}_C\ \text{Bill read } t_1]\!] =$
$[\Box_C R(a) \vee \Box_C R(b)] \wedge \neg[\Box_C R(a) \wedge \Box_C R(b)] \wedge [\Box_C R(a) \leftrightarrow \Box_C R(b)](\Leftrightarrow \bot)$.
b. $[\![O_{\text{EXH-D}}\ O_\sigma\ \text{any book}_D\ \lambda_1\ \text{can}_C\ \text{Bill read } t_1]\!] =$
$[\Diamond_C R(a) \vee \Diamond_C R(b)] \wedge \neg[\Diamond_C R(a) \wedge \Diamond_C R(b)] \wedge [\Diamond_C R(a) \leftrightarrow \Diamond_C R(b)](\Leftrightarrow \bot)$

To explain the acceptability of *any* with possibility modals, the WSA proposes a constraint ('Modal Containment', in (8)) which avoids the derivation of a contradiction for (7b) by exploiting the context-sensitive nature of modals. Consider, for instance, (9a), where we use free variables C and C′ to represent the domain of quantification of the modal. The first conjunct in (9a) collapses the existential component and the domain implicature in (7b).

[2] We will consider only those domain alternatives that correspond to proper subsets of the domain of quantification. A proposition q is an innocently excludable to p, in case every way of conjoining p with as many negated alternatives to p as consistency with p allows for entails $\neg q$ (Fox 2007, Alonso-Ovalle 2008).

The proposition in (9a) is true with respect to the accessible worlds in (9b) when the value of C is $\{w_1, w_2\}$ and that of C′ is $\{w_1\}$.[3]

(8) *Modal Containment*: the modal base in the scalar implicature must be a proper subset of the modal base in the domain implicature. (Chierchia 2013:314)

(9) a. $[\Diamond_C R(a) \wedge \Diamond_C R(b)] \wedge \neg[\Diamond_{C'} R(a) \wedge \Diamond_{C'} R(b)]$

b.

w_1	$R(a) \wedge \neg R(b)$
w_2	$\neg R(a) \wedge R(b)$

We turn now to numeral *any*. The LF of the second sentence in (1a) contains the constituent in (10a), which expresses the proposition that Bill read two or more books. The scalar alternative to (10a), determined by considering a higher value for the numeral, is in the set in (10b). O_σ excludes this alternative, deriving (10c):[4]

(10) a. $[\![$any two books$_D$ λ_1 Bill read $t_1]\!]^g =$
$R(a\oplus b) \vee R(b\oplus c) \vee R(a\oplus c) \vee R(a\oplus b\oplus c)$ $(\Leftrightarrow R(a\oplus b) \vee R(b\oplus c) \vee R(a\oplus c))$
$(g(\mathrm{D}) = [\![\text{books}]\!]^w = \{a, b, c, a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\})$.

b. $\{R(a \oplus b \oplus c)\}$ (scalar alternative) .

c. $[\![O_\sigma$ any two books$_D$ λ_1 Bill read $t_1]\!] =$
$[R(a \oplus b) \vee R(b \oplus c) \vee R(a \oplus c)] \wedge \neg[R(a \oplus b \oplus c)]$

The domain alternatives to (10c), in (11a), yield the pre-exhaustified domain alternatives in (11b).[5] These alternatives are stronger than (10c) and get excluded. The negation of these alternatives, together with the existential component, derives the first conjunct in (11c). Assuming that the extension of the predicate is cumulative, the first conjunct in (11c) entails the negation of the second, yielding a contradiction.

---

[3] In contrast, the first conjunct in (i) entails the negation of the second, regardless of the value of C′.

(i) $[\Box_C R(a) \wedge \Box_C R(b)] \wedge \neg[\Box_{C'} R(a) \wedge \Box_{C'} R(b)]$

[4] We assume that numerals express properties of individuals, where ‘$|x|$’ yields the number of atomic individuals that x consists of: $[\![\text{two}]\!] = \lambda x.|x| \geq 2$.

[5] We ignore any subdomain containing only atomic individuals, since these domains derive a contradiction, which the exhaustifier will negate to no effect. Similarly, the ‘mixed domains’, containing both atomic and non-atomic individuals can be disregarded, since they yield alternatives that are equivalent to those coming from domains containing only plural individuals. In the rest of the paper, we will only consider subdomains closed under sum formation. The alternatives in the first line correspond to the singleton subdomains containing one plural individual. In those cases, the scalar alternative is equivalent to the assertion, and therefore, scalar exhaustification is trivial.

(11) a. $\left\{\begin{array}{l} R(a\oplus b), R(a\oplus c), R(a\oplus c), R(a\oplus b\oplus c),\\ R(a\oplus b)\wedge\neg(R(a\oplus b\oplus c)),\ldots\\ [R(a\oplus b)\vee(R(b\oplus c)]\wedge\neg R(a\oplus b\oplus c)\ldots\end{array}\right\}$

b. $\left\{\begin{array}{l} R(a\oplus b)\wedge\neg R(a\oplus c)\wedge\neg R(b\oplus c), R(a\oplus c)\wedge\neg R(b\oplus c)\wedge\neg R(a\oplus b),\\ R(b\oplus c)\wedge\neg R(a\oplus c)\wedge\neg R(a\oplus b), [R(a\oplus b)\vee R(a\oplus c)]\wedge\neg R(b\oplus c),\\ [R(a\oplus c)\vee R(b\oplus c)]\wedge\neg R(a\oplus b), [R(b\oplus c)\vee R(a\oplus b)]\wedge\neg R(a\oplus c)\end{array}\right\}$

c. $[\![\mathrm{O_{EXH\text{-}D}}\ \mathrm{O}_\sigma$ any two books$_D$ λ_1 Bill read $\mathrm{t}_1]\!] =$
$[R(a\oplus b)\wedge R(b\oplus c)\wedge R(a\oplus c)]\wedge\neg[R(a\oplus b\oplus c)](\Leftrightarrow\bot)$

In contrast, no contradiction is derived in (12a): when $g(\mathrm{C}) = \{w_1, w_2, w_3\}$, the first conjunct in (12a) is true in (12b), where the second conjunct is also true.

(12) a. $[\![\mathrm{O_{EXH\text{-}D}}\ \mathrm{O}_\sigma$ any two books$_D$ λ_1 can$_\mathrm{C}$ Bill read t_1 $]\!] =$
$[\Diamond_\mathrm{C}R(a\oplus b)\wedge\Diamond_\mathrm{C}R(b\oplus c)\wedge\Diamond_\mathrm{C}R(a\oplus c)]\wedge\neg[\Diamond_\mathrm{C}R(a\oplus b\oplus c)](\Leftrightarrow\bot)$.

b.

w_1	$R(a\oplus b)\wedge\neg R(a\oplus b\oplus c)$
w_2	$R(b\oplus c)\wedge\neg R(a\oplus b\oplus c)$
w_2	$R(a\oplus c)\wedge\neg R(a\oplus b\oplus c)$

This setup also derives a contradiction in cases where numeral *any* combines with a necessity modal. Consider (13): collapsing the existential component and the domain implicature that this LF derives gives us the first conjunct in (13), which entails the negation of the second.

(13) $[\![\mathrm{O_{EXH\text{-}D}}\ \mathrm{O}_\sigma$ any two books$_\mathrm{D}$ λ_1 must$_\mathrm{C}$ Bill read $\mathrm{t}_1]\!] =$
$[\Box_\mathrm{C}R(a\oplus b)\wedge\Box_\mathrm{C}R(b\oplus c)\wedge\Box_\mathrm{C}R(a\oplus c)]\wedge\neg[\Box_\mathrm{C}R(a\oplus b\oplus c)](\Leftrightarrow\bot)$

The derivation of a contradiction does not align with the distribution of numeral *any*, then. To explain the attested distribution, the analysis assumes that another interpretation constraint can override the Wide Scope Constraint. Consider again (11c), repeated in (14a). Since the predicate is distributive and cumulative, the first conjunct in (14a) is equivalent to (14b). With a distributive predicate, replacing the numeral in (14a) always yields an existential component and domain implicature equivalent to (14b) (and this meaning component is always inconsistent with the corresponding scalar implicatures). The same is true in (13), where numeral *any* combines with a necessity modal. The situation is different with possibility modals: the first conjunct in (15a) entails (15b), but is not entailed by (15b).

(14) a. $[\![\mathrm{O_{EXH\text{-}D}}\ \mathrm{O}_\sigma$ any two books$_\mathrm{D}$ λ_1 Bill read t_1 $]\!] =$
$[R(a\oplus b)\wedge R(b\oplus c)\wedge R(a\oplus c)]\wedge\neg[R(a\oplus b\oplus c)](\Leftrightarrow\bot)$

b. $R(a)\wedge R(b)\wedge R(c)$

(15) a. $[\![\mathrm{O_{EXH\text{-}D}}\ \mathrm{O}_\sigma$ any two books$_\mathrm{D}$ λ_1 can$_\mathrm{C}$ Bill read t_1 $]\!] =$
$[\Diamond_\mathrm{C}R(a\oplus b)\wedge\Diamond_\mathrm{C}R(b\oplus c)\wedge\Diamond_\mathrm{C}R(a\oplus c)]\wedge\neg[\Diamond_\mathrm{C}R(a\oplus b\oplus c)]$

b. $\Diamond_C R(a) \wedge \Diamond_C R(b) \wedge \Diamond_C R(c)$

In a sense, then, the numeral is redundant in episodic sentences and where *any* combines with a necessity modal. This motivates the Scale Economy Constraint, in (16):

(16) $*$ [O FCI_i ...] if $\text{FCI}_i \in \langle \text{FCI}_1 \ldots \text{FCI}_n \rangle (n > 2)$ and $\forall j (1 \leq j \leq n)$ $[\![$ [O FCI_i ...] $]\!] = [\![$ [O FCI_j ...] $]\!]$ (Chierchia 2013:333)

In sentences with a necessity modal, a violation of the Scale Economy Constraint (and the derivation of a contradiction) can be avoided by letting numeral *any* scope under the modal—at the cost of violating the Wide Scope Constraint. In this case, the resulting meaning is not contradictory, and it does not violate the Scale Economy Constraint. To illustrate, consider (17):

(17) $[\![\text{O}_\sigma \text{ must}_C \text{ any two books}_D\ \lambda_1 \text{ Bill read } t_1]\!] =$
$[\Box_C[R(a \oplus b) \vee R(b \oplus c) \vee R(a \oplus c)]] \ \wedge \neg[\Box_C R(a \oplus b \oplus c)]$

The set containing the negation of the pre-exhaustified domain alternatives to (17) is in (18a) below.[6] If (17) is true, all the antecedents in these conditionals must be false. The strengthened meaning, in (18b), is not a contradiction. It entails that every group of two books is a permitted option for Bill, and is therefore true in the model in (18c). Furthermore, this meaning does not violate the Scale Economy Constraint, since it is not equivalent to the meanings that higher or lower numerals would have given rise to.

(18) a.
$$\left\{ \begin{array}{l} \Box_C R(a \oplus b) \rightarrow [\Box_C(R(b \oplus c) \vee \Box_C(R(a \oplus c)], \\ \Box_C R(a \oplus c) \rightarrow [\Box_C(R(b \oplus c) \vee \Box_C(R(a \oplus b)], \\ \Box_C R(b \oplus c) \rightarrow [\Box_C(R(a \oplus c) \vee \Box_C(R(a \oplus b)], \\ \Box_C[R(a \oplus b) \vee R(b \oplus c)] \rightarrow [\Box_C[R(b \oplus c) \vee R(a \oplus c)] \vee \Box_C[R(a \oplus b) \vee R(a \oplus c)]], \\ \Box_C[R(b \oplus c) \vee R(a \oplus c)] \rightarrow [\Box_C[R(a \oplus b) \vee R(b \oplus c)] \vee \Box_C[R(a \oplus b) \vee R(a \oplus c)]], \\ \Box_C[R(a \oplus b) \vee R(a \oplus c)] \rightarrow [\Box_C[R(a \oplus b) \vee R(b \oplus c)] \vee \Box_C[R(b \oplus c) \vee R(a \oplus c)]] \end{array} \right\}$$

b. $[\![\text{O}_{\text{EXH-D}}\ \text{O}_\sigma \text{ must any two books } \lambda_1 \text{ Bill read } t_1]\!] =$
$[\Box[R(a \oplus b) \vee R(b \oplus c) \vee R(a \oplus c)]] \ \wedge$
$\neg\Box_C[R(a \oplus b) \vee R(b \oplus c)] \wedge$
$\neg\Box_C[R(a \oplus b) \vee R(a \oplus c)] \wedge \neg\Box_C[R(a \oplus c) \vee R(b \oplus c)]$

c.

w_1	$R(a \oplus b) \wedge \neg R(b \oplus c) \wedge \neg R(a \oplus c)$
w_2	$R(b \oplus c) \wedge \neg R(a \oplus b) \wedge \neg R(a \oplus c)$
w_3	$R(a \oplus c) \wedge \neg R(a \oplus b) \wedge \neg R(b \oplus c)$

To summarize: the Wide Scope Constraint and the Scale Economy Condition rule out numeral *any* in episodic sentences, but not in sentences containing a possibility or a necessity modal, as desired. We turn next to the Viability Constraint Analysis.

[6]We exclude from the set below $\neg\Box R(a \oplus b \oplus c)$, since it is entailed by the assertion.

2.2 The Viability Constraint Analysis (Dayal 2013)

Like the WSA, the VCA assumes that *any* is an existential, and that it triggers and excludes pre-exhaustified domain alternatives, but the Wide Scope Constraint, Modal Containment, and Scale Economy Condition constraints are replaced by (19): [7]

(19) *The Viability Constraint:*

1. When *any* does not outscope a modal, each pre-exhaustified domain alternative must be true in the world of evaluation.
2. When *any* outscopes a modal with domain C, each pre-exhaustified domain alternative must be true in the world of evaluation when the domain of the modal is restricted to a subset of C.

The Viability Constraint cannot be satisfied in cases where *any* is in a positive episodic sentence, as in (20a), since, in those cases, the pre-exhaustified domain alternatives, in (20b), are mutually exclusive. The same is true when *any* scopes under a modal.

(20) a. $[\![\text{any book}_\text{D}\ \lambda_1\ \text{Bill read t}_1]\!] = R(a) \vee R(b)$
b. $\{R(a) \wedge \neg R(b), R(b) \wedge \neg R(a)\}$ (PDAs)

When *any* scopes over a possibility modal, as in (21a), the Viability Constraint can be satisfied. When C = $\{w_1, w_2\}$, (21a) will be true in (21d). The same is true for (21c), the result of strengthening (21a) with the negation of the pre-exhaustified alternatives. At the same time, each pre-exhaustified domain alternative can be true when the domain of its modal is a subset of C, if we let g(C′) = $\{w_1\}$ and g(C″) = $\{w_2\}$.

(21) a. $[\![\text{any book}_\text{D}\ \lambda_1\ \text{can}_\text{C}\ \text{Bill read t}_1]\!] = \Diamond_\text{C} R(a) \vee \Diamond_\text{C} R(b)$
b. $\{\Diamond_{\text{C}'} R(a) \wedge \neg\Diamond_{\text{C}'} R(b), \Diamond_{\text{C}''} R(b) \wedge \neg\Diamond_{\text{C}''} R(a)\}$ (PDAs)
c. $\Diamond_\text{C} R(a) \wedge \Diamond_\text{C} R(b)$
d.

w_1	$R(a) \wedge \neg R(b)$	w_2	$\neg R(a) \wedge R(b)$

The situation changes when *any* scopes over a necessity modal. When the assertion in (22a) is strengthened with the negation of each pre-exhaustified alternative in (22b), we end up with the conjunction in (22c). If (22c) is true, then, no pre-exhaustified domain alternative will be true when its modal domain is the value of C or a subset of it.

(22) a. $[\![\text{any book}_\text{D}\ \lambda_1\ \text{must}_\text{C}\ \text{Bill read t}_1]\!] = \Box_\text{C} R(a) \vee \Box_\text{C} R(b)$
b. $\{\Box_\text{C} R(a) \wedge \neg\Box_\text{C} R(b), \Box_\text{C} R(b) \wedge \neg\Box_\text{C} R(a)\}$ (PDAs)
c. $\Box_\text{C} R(a) \wedge \Box_\text{C} R(b)$

To account for the difference between *any* and numeral *any*, the VCA—like the WSA—assumes that the existential component of *any* scopes under the modal. Unlike the WSA, the

[7] The wording of the constraint is ours.

VCA takes numeral *any* to introduce two existentials: one corresponding to the numeral (a generalized quantifier ranging over degrees, (23)) and the other to *any*. In (25), the numeral moves from a DP-internal position (the sister of a covert MANY (24)) over the modal.

(23) $[\![\text{two}]\!] = \lambda P_{\langle d,t\rangle}.\exists d[d = 2 \wedge P(d)]$ (24) $[\![\text{MANY}]\!] = \lambda d.\lambda x.|x| = d$

(25) $[\![\text{two } \lambda_2 \text{ must}_C \text{ [any t}_2\text{-MANY books}_D \text{] } \lambda_1 \text{ Bill reads t}_1]\!] =$
$\Box_C[R(a \oplus b) \vee R(b \oplus c) \vee R(a \oplus c)]$

The truth-conditions for (25) correspond to those that we get for narrow scope numeral *any* under the WSA (excluding exhaustification.) We also get the same pre-exhaustified alternatives. The Viability Condition is checked at the smallest constituent containing every component of the free choice item, so, in this case, it is checked at the topmost node, since *any* and the numeral form a complex free choice item. When the assertion is strengthened with the exclusion of the pre-exhaustified alternatives, the Viability Constraint can be satisfied in models like (18c), accounting for the acceptability of numeral *any* with necessity modals.

3. Collective predicates

Both the WSA and the VCA capture the distribution and interpretation of *any* DPs, as reported in the introduction. So there is little empirical motivation to favor one over the other. All the observations discussed so far have concerned sentences containing distributive predicates. In this section, we will turn our attention to sentences containing collective predicates, where the predictions made by the two analyses differ: while the VCA correctly predicts the attested interpretations, the WSA, as is, overgenerates.

3.1 The observation

Consider the sentence in (26) below, which contains the collective predicate *mix*:[8]

(26) John must mix any two drinks.

Our informants judge (26) as true in the context in (27b) and false in the context in (27a). The reported intuition is that John will meet his requirement if he simply mixes a single group of two drinks. Under this interpretation, the sentence in (26) conveys two meaning components: (i) that for each group of two drinks x, John is permitted to mix x, and (ii) that John is required to mix a group of two drinks but not required to mix any particular group. These two claims together make up what we call the *universal permission + existential requirement* reading.

[8] We are interested in the reading where two drinks are being mixed together, not the reading where a single drink can be mixed.

(27) a. *Bar tender competition (I). Do all cocktails.* There is coke, whiskey, and gin. John is required to mix coke and whiskey, coke and gin, and whiskey and gin

b. *Bar tender competition (II). Choose your cocktail.* Same drinks. John is permitted to mix any couple of drinks. He is required to mix at least one pair, but not required to mix any particular pair.

We will now assess the predictions of the WSA and the VCA with respect to this data point.

3.2 Assessing the predictions

3.2.1 The Wide Scope Constraint Analysis

The WSA makes opposite predictions to those reported: it predicts that (26) should be true in (27a) and false in (27b). Under the WSA, the sentence in (26) has the LF in (28a) below:

(28) a. $[\![\mathrm{O_{EXH\text{-}D}}\ \mathrm{O}_\sigma$ any two drinks$_\mathrm{D}$ λ_1 must$_\mathrm{C}$ John mix $\mathrm{t}_1]\!] =$
$$\underbrace{[\Box_C M(a \oplus b) \wedge \Box_C M(b \oplus c) \wedge \Box_C M(a \oplus c)]}_{\text{assertion + domain implicature}} \wedge \underbrace{\neg[\Box_C M(a \oplus b \oplus c)]}_{\text{scalar implicature}}$$

b. $\Box_C M(a \oplus b \oplus c)$

The LF in (28a) satisfies the Wide Scope Constraint. Nothing enforces a violation of the constraint. In particular, the Scale Economy Constraint is not violated: because the predicate is collective, quantifying over groups of drinks containing more or less drinks would have yielded a different meaning. In a model like (29c), where John only ever mixes groups of exactly two drinks in any permitted world, (29a) would be true but (29b) would not.

(29) a. John must mix any two drinks. $\rightsquigarrow$ John is required to mix all pairs of drinks and he is not required to mix a larger group of drinks.

b. John must mix any three drinks. $\rightsquigarrow$ John is required to mix all groups of three drinks and he is not required to mix a larger group of drinks.

c.

w_1	$M(a \oplus b) \wedge \neg M(b \oplus c) \wedge \neg M(a \oplus c)$
w_2	$M(b \oplus c) \wedge \neg M(a \oplus b) \wedge \neg M(a \oplus c)$
w_3	$M(a \oplus c) \wedge \neg M(a \oplus b) \wedge \neg M(b \oplus c)$

Because the predicate is collective, the proposition in (28a) is not a contradiction: the first conjunct in (28a) does not entail the scalar alternative in (28b) and so it is compatible with the second conjunct in (28a). The proposition conveys that John is required to mix every group of two drinks and that he is not required to mix any larger group of drinks. This proposition is true in (27a) and false in (27b), contrary to what our informants report.

3.2.2 The Viability Constraint Analysis

In contrast with the WSA, the VCA predicts the judgements reported above. To illustrate, consider the LF of the sentence in (26), in (30).

(30) LF: two$_2$ must$_C$ any t$_2$ drinks$_D$ λ_1 John mix t$_1$

This LF denotes the proposition in (31), which conveys that in every world, there is at least one group of two drinks which gets mixed. The Viability Constraint requires all pre-exhaustified domain alternatives in (32) to be true when the domain of their modal is restricted to some subset of C. This means that (30) must exclude models where John is required to mix all pairs of drinks, as in (33).

(31) $\Box_C[M(a \oplus b) \lor M(b \oplus c) \lor M(a \oplus c)]$ (Assertion)

(32)
$$\left\{\begin{array}{l} \Box_C M(a \oplus b) \land \neg\Box_C[M(b \oplus c) \lor M(a \oplus c)], \\ \Box_C M(a \oplus c) \land \neg\Box_C[M(b \oplus c) \lor M(a \oplus b)], \\ \Box_C M(b \oplus c) \land \neg\Box_C[M(a \oplus c) \lor M(a \oplus b)], \\ \Box_C[M(a \oplus b) \lor M(b \oplus c)] \land \neg\Box_C[M(b \oplus c) \lor M(a \oplus c)] \land \neg\Box_C[M(a \oplus b) \lor M(a \oplus c)], \\ \Box_C[M(b \oplus c) \lor M(a \oplus c)] \land \neg\Box_C[M(a \oplus b) \lor M(b \oplus c)] \land \neg\Box_C[M(a \oplus b) \lor M(a \oplus c)], \\ \Box_C[M(a \oplus b) \lor M(a \oplus c)] \land \neg\Box_C[M(a \oplus b) \lor M(b \oplus c)] \land \neg\Box_C[M(b \oplus c) \lor M(a \oplus c)] \end{array}\right\}$$

(33)

w_1	$M(a \oplus b) \land M(b \oplus c) \land M(a \oplus c)$
w_2	$M(a \oplus b) \land M(b \oplus c) \land M(a \oplus c)$

To see why: consider the first three pre-exhaustified domain alternatives in (32): none of them can be true in any subdomain of (33). The target sentence is then predicted to be rejected in the context in (27a), repeated in (34) below.

(34) *Bar tender competition (I). Do all cocktails.* There is coke, whiskey, and gin. John is required to mix coke and whiskey, coke and gin, and whiskey and gin

At the same time, strengthening the meaning in (31) with the negation of the pre-exhaustified domain alternatives in (32) allows for models like (35) below, where John is required to mix some couple of drinks or other, and where the Viability Constraint is satisfied.

(35)

w_1	$M(a \oplus b) \land \neg M(b \oplus c) \land \neg M(a \oplus c)$
w_2	$M(b \oplus c) \land \neg M(a \oplus b) \land \neg M(a \oplus c)$
w_3	$M(a \oplus c) \land \neg M(a \oplus b) \land \neg M(b \oplus c)$

We then expect the target sentence to be true in the scenario in (27b), repeated in (36) below, in agreement with the judgments of our informants.

(36) *Bar tender competition (II). Choose your cocktail.* Same drinks. John is permitted to mix any couple of drinks. He is required to mix at least one pair, but not required to mix any particular pair.

3.2.3 Summary: assessment

Informally, we can see that the Scale Economy Constraint is sensitive to the type of predicate that *any* combines with. In particular, it forces *any* to scope under the modal, in violation of the Wide Scope Constraint, when it combines with distributive predicates. The extension to collective predicates poses a challenge, because, in those cases, nothing forces a violation of the Wide Scope Constraint. This challenge concerns the status of the Scale Economy Constraint, which attributes narrow scope numeral *any* to the richness of the scale. Sentences containing collective predicates illustrate scenarios where the numeral is no longer pathological and yet numeral *any* still receives a narrow scope interpretation. On the other hand, the decomposition of the complex free choice item, numeral + *any*, under the VCA, is insensitive to predicate type: under that approach *any* scopes under the modal irrespective of the predicate type that it combines with. This leads to the VCA straightforwardly capturing the attested judgments for the interaction between numeral *any* and collective predicates.[9]

4. To conclude

We have seen that in many instances, both the WSA and the VCA mirror each other in capturing the distribution of *any* and numeral *any*. However, these two analyses diverge in predictions when we look at sentences containing numeral *any* with collective predicates. In these cases, we observe that only the VCA analysis makes the right predictions.

References

Alonso-Ovalle, Luis. 2008. Innocent exclusion in an alternative semantics. *Natural Language Semantics* 16:115–128.

Chierchia, Gennaro. 2013. *Logic in Grammar*. Oxford: Oxford University Press.

Dayal, Veneeta. 2005. The universal force of free choice *any*. *Linguistic Variation Yearbook* 4:5–40.

Dayal, Veneeta. 2013. A viability constraint on alternatives for free choice. In *Alternatives in Semantics*, ed. by Anamaria Fălăuş, 88–122. Palgrave Macmillan.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In *Presupposition and implicature in compositional semantics*, 71–120. Springer.

Luis Alonso-Ovalle and Jonathan Palucci
luis.alonso-ovalle@mcgill.ca
jonathan.palucci@mail.mcgill.ca

[9]There could still be a wide scope reading for sentences containing a necessity modal and numeral *any*.